Counterfactual Generative Models for Time-Varying Treatments

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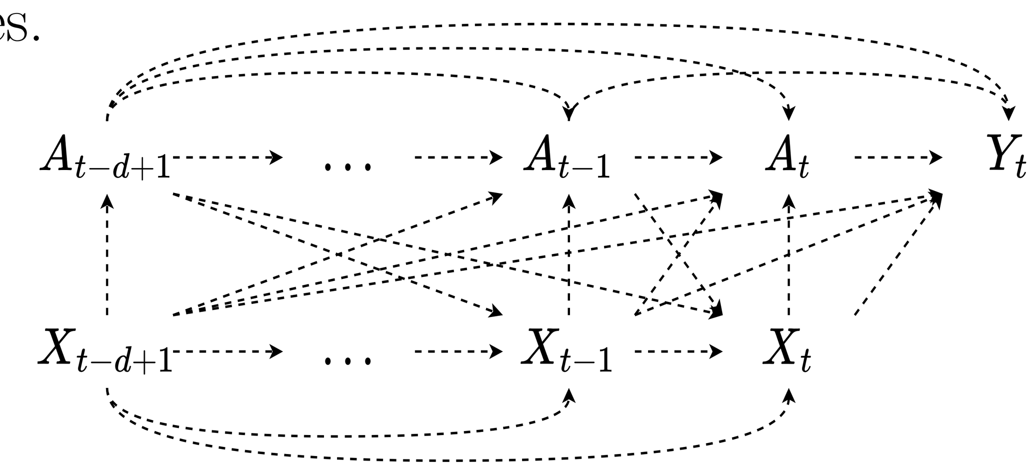
Introduction

Goal: Our goal is to systematically address the following three practical challenges for data-driven decision making in one versatile and model-agnostic framework.

- **Counterfactual Inference:** The goal is to infer what would have happened if were to act in a way *not* observed in previous results.
- **Temporal Setting:** Collected data is blurred with treatments and confounders that has *time-dependent* structures.



- **Distribution Learning:** People care about the entire counterfactual *distribution* of the outcome variable.

Notation: At time t, denote the outcome variable as Y_t, denote the d-length history of treatments and covariates as $\overline{A}_t = (A_{t-d+1}, \dots, A_t)$ and $\overline{X}_t = (X_{t-d+1}, \dots, X_t)$. Lowercase letters represents their realizations. We use f to denote distribution.

Proposed Method

Learning Objective: We aim to minimize the Kullback–Leibler (KL) divergence between a proxy conditional distribution $f_\theta(\cdot|a)$ and $f_{\bar{a}}$. The learned model will be a generator function denoted as

$$g_\theta(z, \overline{a}) : \mathbb{R}^r \times \mathcal{A}^d \to \mathcal{Y}$$

Loss Function: Theoretical result derived in or paper shows that

$$f_{\bar{a}}(y) = \int \frac{\mathbb{1}\{\overline{A} = \overline{a}\}}{\prod_{\tau=t-d}^{t} f\left(A_\tau|\overline{A}_{\tau-1}, \overline{X}_\tau\right)} f\left(y, \overline{A}, \overline{X}\right) d\overline{A} d\overline{X},$$

Using this lemma, we can approximate the generative learning objective by maximizing the log-likelihood:

$$\mathbb{E}_{y \sim f_{\bar{a}}} \log f_\theta(y|\overline{a}) \approx \sum_{(y,\overline{a},\overline{x}) \in \mathcal{D}} w_\phi(\overline{a}, \overline{x}) \log f_\theta(y|\overline{a}),$$

where $w_\phi(\overline{a}, \overline{x})$ denotes the subject-specific IPTW, parameterized by $\phi \in \Phi$, which takes the form:

$$w_\phi(\overline{a}, \overline{x}) = \frac{1}{\prod_{\tau=t-d}^{t} f_\phi(a_\tau|\overline{a}_{\tau-1}, \overline{x}_\tau)}.$$

Model: Our proposed model, `MSCVAE`, adopts an encoder-decoder structure. Figure 1 shows the learning objective of the model, and Figure 2 shows the model architecture of `MSCVAE`.

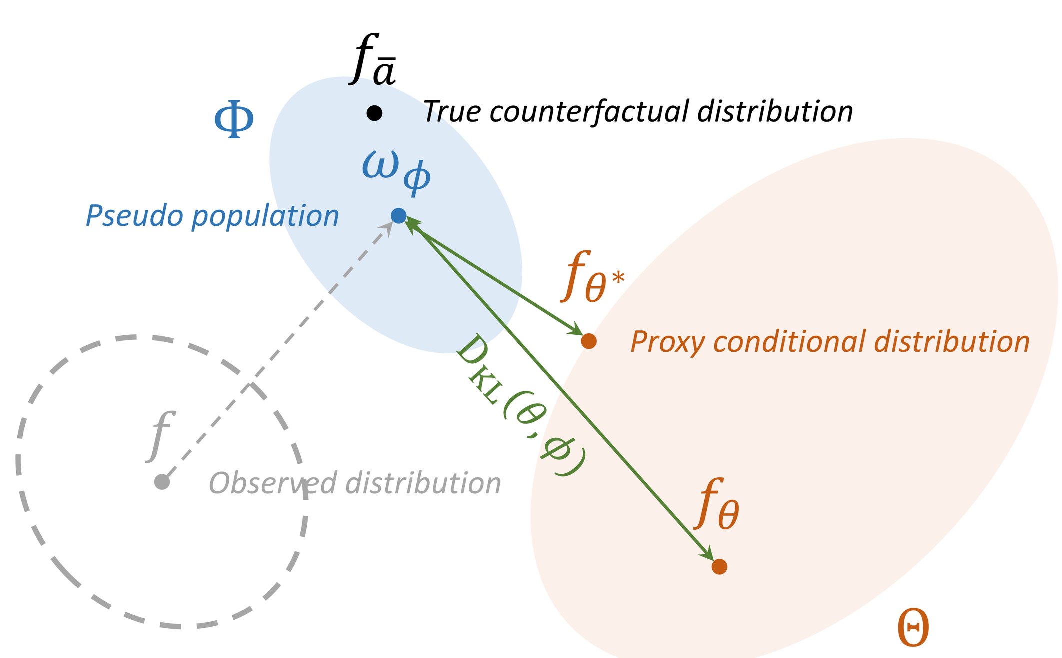


Figure 1: Learning Objective

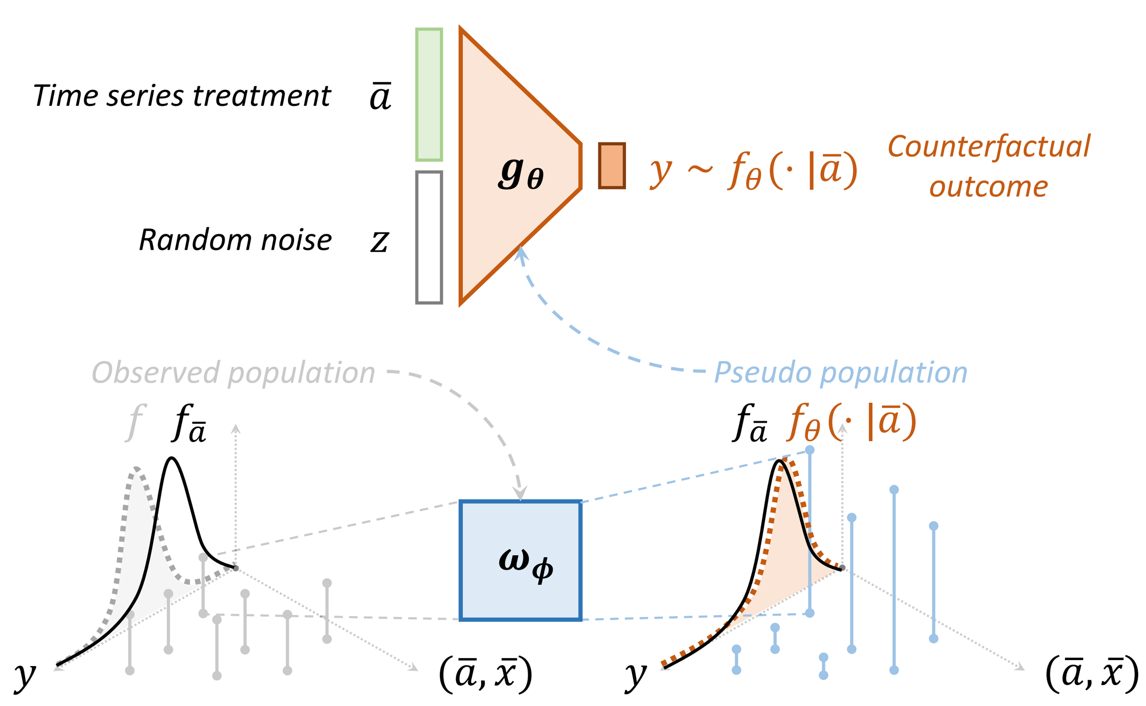
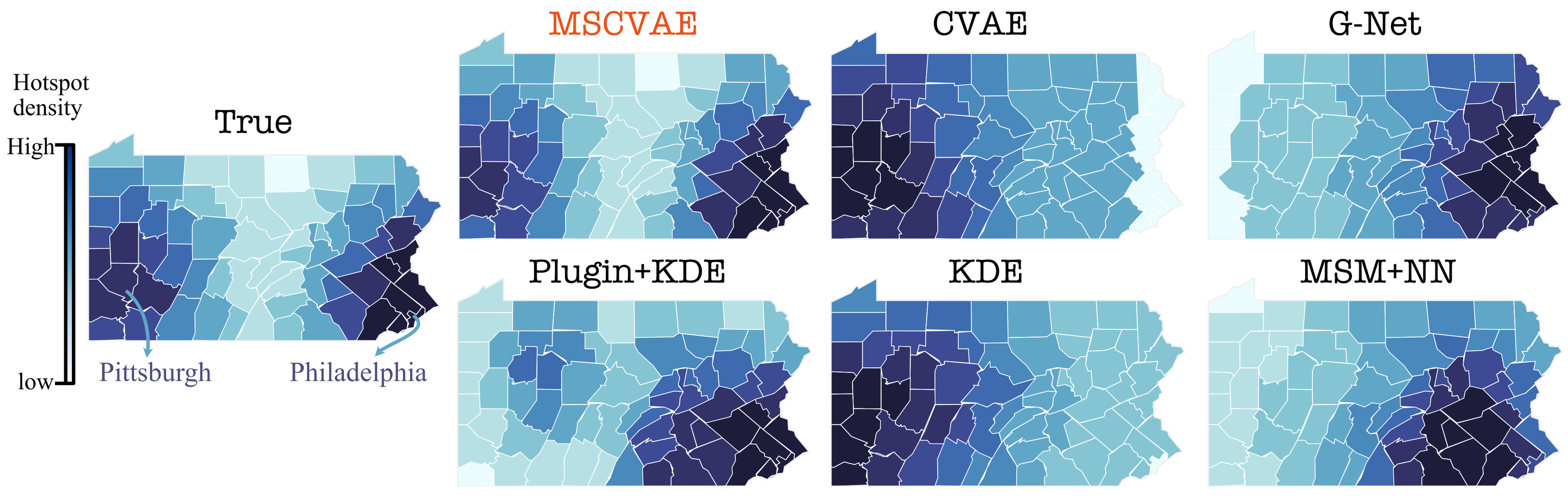


Figure 2: Model Architecture

Evaluation

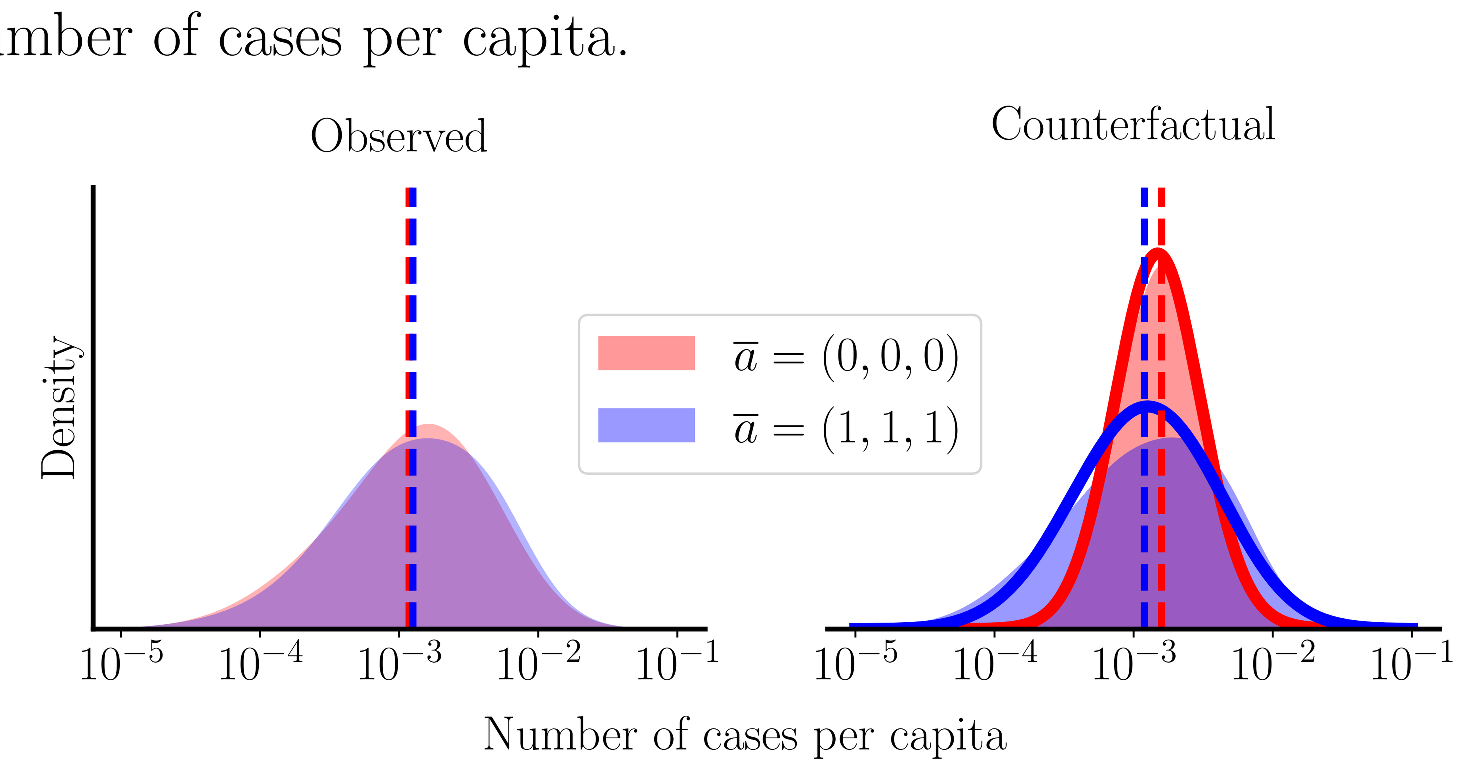
Synthetic Experiment: Our model demonstrates superior performance both quantitatively and visually compared to existing baselines. For synthetic data, we evaluate measures of distance between generated and true counterfactual distribution. For semi-synthetic data constructed from COVID-19 data, we can visually compare their distributional resemblance.

	$d=1$		$d=3$		$d=5$	
Methods	Mean	Wasserstein	Mean	Wasserstein	Mean	Wasserstein
`Linear MSM`	**0.003**	NA	0.055	NA	0.186	NA
`KDE`	0.246	0.433	0.528	0.579	0.536	0.601
`IPTW+KDE`	0.010	0.127	0.048	0.133	**0.146**	0.181
`CVAE`	0.263	0.264	0.524	0.559	0.537	0.612
`MSCVAE`	0.008	**0.053**	**0.043**	**0.107**	0.147	**0.171**



Case Study : COVID-19

Description: 5 features of 3219 U.S. counties are collected in 2020-2021 spanning across 49 weeks. We aim to make counterfactual predictions regarding how mask policies affect COVID-19 number of cases per capita.



Insight: Imposing mask mandate can decrease the **mean** of the distribution, but increases its **variance** in the same time. This implies that while mask mandate tend to help control virus spread, a thorough examination of the specific circumstances is highly recommended for mask-policymakers to avoid any unintended consequences.